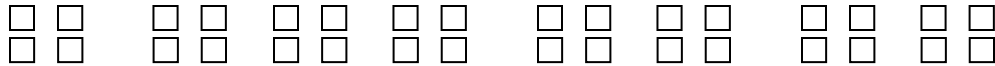# MATH 211 FINAL EXAM REVIEW PROBLEMS with ANSWERS

1. $32 \div 4$ in the sharing interpretation of division, base ten pieces:

Share among 4 groups – there are 8 in each group so $32 \div 4 = 8$.

2. $32 \div 4$ in the measurement interpretation of division, base ten pieces

Make groups of size 4 – there are 8 groups so $32 \div 4 = 8$.

3. Write a short and simple story problem for each:
   a. Illustrating $18 \div 6$ for the sharing concept of division.

I have 18 apples and want to share them among 6 friends. How many apples does each friend get?

   b. Illustrating $18 \div 6$ for the measurement concept of division.

I have 18 apples and I want to put them into bags with 6 in each bag. How many bags do I need?

   c. Illustrating $12 - 7$ for the take away concept of subtraction

I have 12 apples and my brother takes 7 apples. How many do I have left?

   d. Illustrating $12 - 7$ for the comparison concept of subtraction

I have 12 apples and my brother has 7 apples. How many more apples do I have?

   e. Illustrating $12 - 7$ for the missing addend concept of subtraction

I have 7 apples, but I need 12 to make a pie. How many more apples do I need?

4. The number 354,109,373,276,4x0 will be divisible by 6 if x is replaced by_____?
0 or 3 or 6 or 9

5. The Uris satellite circles the earth every 308 hours. The Arub satellite circles the earth every 660 hours. If both satellites were above Monroe, Louisiana at 7 AM on April 12, the earliest time they will both again be above Monroe is in this many hours:
LCM(308,660) = 4620 = 2x2x3x5x7x11 hours

6. Which one of the following pairs of numbers is relatively prime?
      (10, 20), (23, 46), (16, 30), (15, 42), (32, 125)
GCF(32,125)=1 so (32,125) are relatively prime

7. For this problem: Choose all, if any, that are correct.
The number 354,109,373,286,460 is divisible by: 2, 3, 4, 5, 6, 9, 10, 11?
It is divisible by 2,4,5,10,11

8. Find the LCM (1125, 70) using any method (no calculator).
LCM(1125,70) = 2 x 3^2 x 5^3 x 7 = 15750

9. Find the GCF (1125, 70) using any method (no calculator).
GCB(1125,70) = 5

10. What is the greatest prime that must be checked to determine if 179 is prime or composite?
13: The square root of 179 is ~ 13.37 so 13 is the largest prime smaller than the square root.

11. GCF(x, y) = 10. LCM(x, y) = 900. x < y < 150. Find x and y.
x = 2 x 5 x 3 x 3 = 90; y = 2 x 5 x 2 x 5 = 100

12. Explain why $2^2{\times}3^2{\times}15$ is not a prime factorization of 540.
15 is not prime

13. If a number is not divisible by 6, can it be divisible by 3? Explain.
Yes - for example 9 is not divisible by 6 but it is divisible by 3.

14. If a number is not divisible by 3, can it be divisible by 6? Explain.
No, 3 divides 6 so if a number is not divisible by 3 it can't be divisible by 6.

15. If a number is not divisible by 6, can it be divisible by 9? Explain.
Yes, for example 9 is not divisible by 6, but it is divisible by 9

16. If a number is not divisible by 2, can it be divisible by 4? Explain.
No, 2 divides 4 so if a number is not divisible by 2 it can't be divisible by 4.

17. If a number is not divisible by 4, can it be divisible by 2? Explain.
Yes, for example 6 is not divisible by 4 but it is divisible by 2.

18. Use black and red tile models with R for red tiles and B for black tiles to show the following: Write clearly and explain your work.

a. 7 + (- 5) **BBBBBBB**
      RRRRR
the 5 black and red cancel leaving 2 black
Answer: 2

b. 4 - 6   **BBBB**
add two zero pairs to get
BBBBBB
RR
take away 6 black to get
RR Answer: -2

c. 3 - (- 1)
BBB add in a zero pair to get BBBBR take out one red leaving BBBB
Answer: 4

d. 2 × - 4 **two times, put in 4 red tiles:**
RRRR RRRR
Answer: -8

SOLUTIONS: Math 211 Final Practice Problems, Page 2

e. -2 × - 4

Start with some zero pairs;
BBBBBBBB
RRRRRRRR
then 2 times take out 4 red
leaving BBBBBBBB
Answer: 8

f. -2 × 4

Start with some zero pairs;
BBBBBBBB
RRRRRRRR
then two times take out 4 black leaving
RRRRRRRR
Answer: -8

g. - 9 ÷ 3

Put 9 red tiles into 3 groups. How many in each group? 3 red
RRR RRR RRR
Answer -3

h. - 9 ÷ -3

Put 9 red tiles into groups of 3 red tiles each. How many groups? 3 groups
RRR RRR RRR
Answer: 3

19. Closed or not?
    a. The set of whole numbers for division. **NO (e.g. 3 ÷ 2 is not a whole number)**
    b. The set of whole numbers for addition. **YES**
    c. The set of whole numbers for subtraction. **NO (e.g. 2-7 is not a whole number)**
    d. The set of even whole numbers for multiplication. **YES**
    e. The set of integers for multiplication. **YES**
    f. The set of integers for division. **NO (e.g. 5 ÷ 2 is not an integer)**
    g. The set of negative integers for addition. **YES**
    h. The set of positive integers for subtraction. **NO (e.g. 1-3 is not positive)**
    i. The set of even integers for subtraction. **YES**
    j. The set of odd integers for subtraction. **NO (e.g. 5 – 3 is not odd)**
    k. The set of {0, 1} for addition **NO (e.g. 1 + 1 is not in the set)**
    l. The set of {0, 1} for multiplication **YES**

20. Commutative or not?
    a. The set of whole numbers for division. **NO (e.g. 5 ÷ 2 ≠ 2 ÷ 5)**
    b. The set of whole numbers for addition. **YES**
    c. The set of whole numbers for subtraction. **NO (e.g. 5 - 2 ≠ 2 - 5)**
    d. The set of integers for multiplication **YES**
    e. The set of integers for division. **NO (e.g. 5 ÷ 2 ≠ 2 ÷ 5)**
    f. The set of negative integers for addition. **YES**
    g. The set of even integers for subtraction. **NO (e.g. 4-2 ≠ 2-4)**
    h. The set {0, 1} for addition **YES**
    i. The set {0, 1} for multiplication **YES**

21. Associative or not? **(a+b) + c = a + (b+c)**
    a. The set of whole numbers for division. **NO**
    b. The set of whole numbers for addition. **YES**
    c. The set of whole numbers for subtraction. **NO**
    d. The set of integers for multiplication. **YES**
    e. The set of negative integers for addition. **YES**
    f. The set of even integers for subtraction. **NO**

22. Identity
    a. What is the identity for whole numbers for addition? For integers? **0**
    b. What is the identity for whole numbers for multiplication? For integers? **1**

23. Distributive
    a. What is the distributive property for whole numbers for multiplication over addition? For integers? For multiplication over subtraction?
    multiplication over addition (whole numbers & integers):
        a x (b+c) = (a x b) + (a x c)
    multiplication over subtraction (whole numbers & integers):
        a x (b-c) = (a x b) – (a x c)

    b. What is the distributive property for integers for multiplication subtraction?
                        (see previous problem)

24. Valid or invalid?
    All children love to draw.
    <u>Cindy is a child</u>.
    Therefore, Cindy loves to draw.   **VALID**

25. Valid or invalid?
    Some educated people are rascals.
    <u>Doctors are educated people.</u>
    Therefore, doctors are not rascals.   **INVALID**
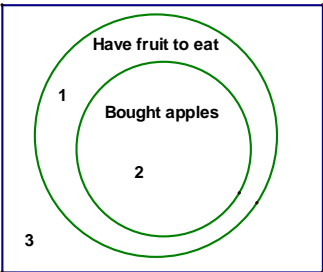
26. List the factors of 12. List the first 4 multiples of 12.
    Factors of 12: 1,2,3,4,6,12; first 4 multiples of 12: 12, 24, 36, 48
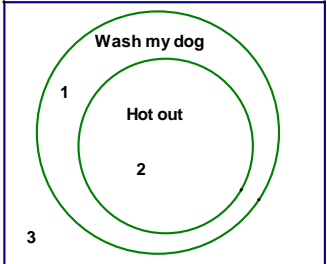
27. Rewrite each of the following using i) converse, ii) inverse and iii) contrapositive. In each case use a Venn diagram to show whether the new statement is valid or invalid.

    a. If I buy apples then I have fruit to eat.

VENN DIAGRAM	**CONVERSE: If I have to fruit to eat, then I bought apples. Invalid – At #1 I have fruit to eat, but didn't buy apples.**
Have fruit to eat, 1, Bought apples, 2, 3	**INVERSE: If I do not buy apples, then I do not have fruit to eat. INVALID At #1 – I did not buy apples, but still have fruit to eat**
	CONTRAPOSITIVE: If I do not have fruit to eat, then I did not buy apples. VALID If I did not have fruit I must be at #3 which means I did not buy apples.

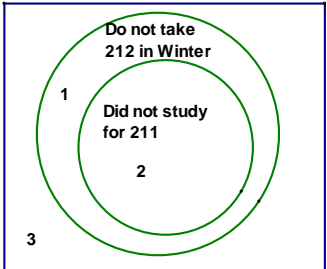b. I will wash my dog if it is hot out.
Rewrite as If P then Q: If it is hot out, then I will wash my dog.

VENN DIAGRAM	CONVERSE: If I wash my dog, then it is hot out. INVALID – at #1 I wash my dog, but it is not hot out. INVERSE: If it is not hot out, then I do not wash my dog: INVALID – at #1 it is not hot out, but I still wash my dog. CONTRAPOSITIVE: If I do not wash my dog, then it is not hot out. VALID: If I don't wash my dog, I must be at #3 which means I it is not hot out.
Wash my dog / 1 / Hot out / 2 / 3	

c. I will not take Math 212 in the winter if I don't study for the math 211 final.

Rewrite as IF P THEN Q:
If I don't study for the math 211 final, then I will not take Math 212 in the winter.

VENN DIAGRAM	CONVERSE: If I don't take Math 212 in the winter, then I didn't study for my 211 final. INVALID: At #1 I do not take 212 in the winter, but I did study for the 211 final. INVERSE: If I study for the math 211 final, then I will take math 212 in the winter: INVALID: AT #1 I studied for the 211 final, but still don't take 212 in the winter. CONTRAPOSITIVE: If I take math 212 in the winter, then I studied for the 211 final. VALID: If I take 212 then I must be at #3 in which case I did study for my 211 final.
Do not take 212 in Winter / 1 / Did not study for 211 / 2 / 3	

28. Write 1247_{ten} in expanded form (base 10).
$(1 \times 10^3) + (2 \times 10^2) + (4 \times 10) + 7$

29. How many units are in 1847_{nine}?
$(1 \times 9^3) + (8 \times 9^2) + (4 \times 9) + 7 = 1420$

30. Convert 184700_{ten} to base sixty.
$(51 \ \ 18 \ \ 20)_{sixty}$

31. What are the digits in any base b?
$0, 1, 2, \ldots, b-1$

32. What are the place values in any base b?
$1, b^2, b^3, b^4, \ldots$

33. Sketch the base four number pieces representing this addition, including all regroupings. Show the addition algorithm and record the resulting base four numeral.
$2311_{four} + 203_{four}$
Answer: 3120_{four} (drawings/algorithm not shown)

34. Sketch the base four number pieces representing this subtraction, including all regroupings. Show the subtraction algorithm and record the resulting base four numeral.

$$222_{four} - 133_{four}$$

Answer: 23$_{four}$ (drawings/algorithm not shown)

35. Sketch the base four number pieces representing this multiplication; including all regroupings. Show the multiplication algorithm and record the resulting base four numeral.

$$222_{four} \times 133_{four}$$

Answer: 110112$_{four}$ (drawings/algorithm not shown) Note: this is too big. Try this problem instead: 22$_{four}$ x 13$_{four}$ – answer is 1012$_{four}$.

36. Select 4 flats, 6 longs, and 2 units from your base ten pieces. Using <u>only</u> these pieces (all of them), and making no exchanges, form a rectangle.

Neatly sketch the rectangle you made, label the edge dimensions and the four partial products and show the final product it represents.
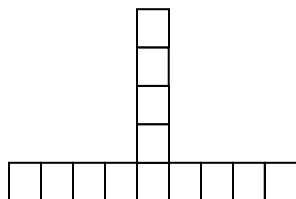**Answer**:


```
       22
     x 21
        2 (1 x 2)
       20 (1 x 20)
       40 (20 x 2)
      400 (20 x 20)
      462
```
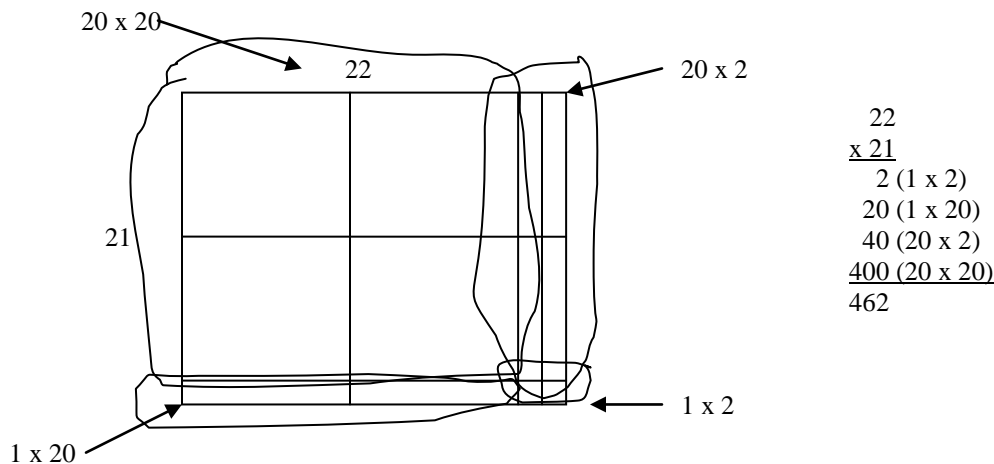
37. Study the pattern below.
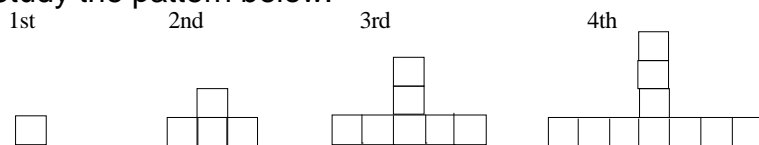
a. If this pattern of tiles continues, draw the 5th figure.

b. If this pattern of tiles is extended to the 150th figure, describe the 150th figure.
The 150th figure will be shaped like an upside down T with 2(150)-1 = 299 tiles along the bottom and 149 tiles stacked on top of the middle bottom tile.

38. The following sequence of figures begins repeating in the fifth figure.
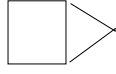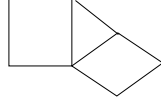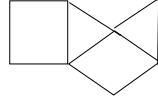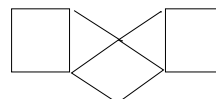

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

    a. Describe and draw the 6th figure.
Add a triangle to the right of the 5th figure

    b. How many triangles will there be in the 163rd, the 164th and the 166th figures?
       Explain clearly for credit, a long list of numbers will receive no credit.
Figure 163 will have 81 triangles.
Figure 164 will have 82 triangles.
Figure 166 will have 83 triangles.

39. Arithmetic, geometric and/or finite differences (1st or 2nd)?
Find a pattern in the following sequence and write the next two terms of the sequence.
      2, 5, 8, 11, 14, … **Answer: 17,20 (arithmetic & finite differences)**

40. Arithmetic, geometric and/or finite differences (1st or 2nd)?
Find a pattern in the following sequence and write the next two terms of the sequence.
      2, 5, 12, 24, 42, …. **Answer: 67,100 (finite differences)**

41. Arithmetic, geometric and/or finite differences (1st or 2nd)?
Find a pattern in the following sequence and write the next two terms of the sequence.
      3, 12, 48, 192, … **Answer: 768, 3072 (geometric)**

42. Arithmetic, geometric and/or finite differences (1st or 2nd)?
Find a pattern in the following sequence and write the next two terms of the sequence.
      0, 1, 7, 18, 34, … **Answer: 55, 81 (finite differences)**

43. Circle to indicate if each statement is true or false. Explain.
    Let: Universal Set = {5, 6, 7, 8, 9, 10} A = {5, 6, 9} B = {5, 6} C = {7, 8, 9}

			Explain
a.	F	$A \subseteq B$	**9 is in A, but not B**
b.	T	$5 \in B$	**5 is an element of B**
c.	F	$B \subset B$	**B is not a PROPER subset of itself**
d.	T	$(A \cup C)' = \{10\}$	**A $\cup$ C={5,6,7,8,9} so the complement is just {10}**
e.	F	B = C	**The sets do not have the same elements.**
f.	T	$A \cap B = \{5, 5, 6, 6\}$	**But it is more proper just to write A $\cap$ B={5,6} (without the duplicates)**

44. Using your attribute piece set, let various sets be A, B, C etc. and describe: **Answers will vary depending on what you choose for A,B,C. For example, one possibility is: A = Blue, B= circles, C = large, then the answers would be:**

  a. $A \cup B$ **= Any piece that is blue or circular**

  b. $A \cap B$ **= Blue circles**

  c. $(A \cup C)'$ **= Any large piece that is not blue**

  d. $A \cap B \cap C$ **= large, blue circles**

  e. $(A \cap B \cap C)'$ **=Any piece except small, blue circles**

  f. $(A \cup B \cup C)'$ **= Small pieces that are red or yellow and not circular**

  g. Describe two sets so that $A \cap B = \varnothing$ **A = blue, B= red**

45. Determine the following:

  a. $6 \div 2 \times 3 + (4 - 1)^2$ **Answer: 18**

  b. $4 \times (3+1) - 2^4$ **Answer: 0**

  c. $18 - 3 \times 2 \div 2 + 7$ **Answer: 22**

  d. $12 + 7 - 8 \div 4 - 1 \times 7$ **Answer: 10**

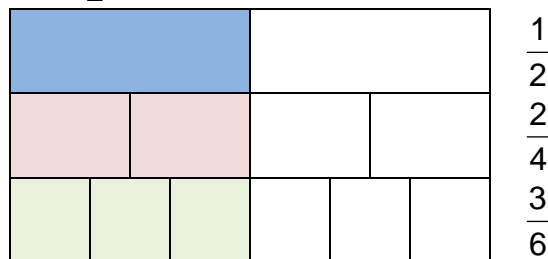46. Use Polya's four steps for problem solving to solve the following:
  a. A farmer is building a fence in the shape of a rectangle of dimensions 30 yards by 40 yards. There is a fence post in every corner and one every two yards. How many fence posts will he use?
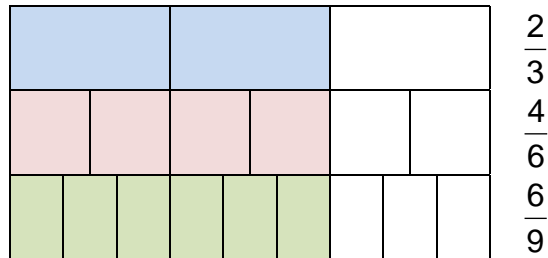   **Answer: He will use 70 fence posts.**

  b. Jill's mother gave her some money to go shopping. Jill spent half the money on a new pair of shoes, then she spent $10 on a CD. After that she spent half of what was left over on lunch and had $12 left. How much money did her mother give her?
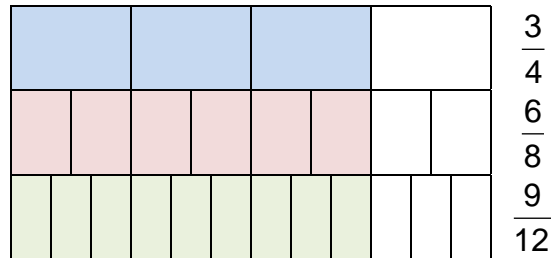   **Answer: Jill's mother gave her $68 to go shopping.**
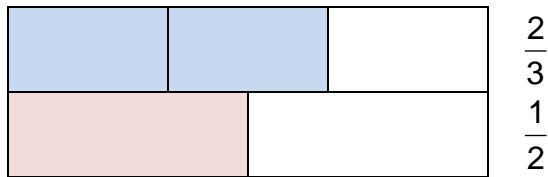
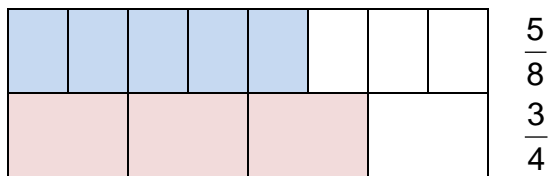47. 3 Fractions equivalent to $\dfrac{1}{2}$

$\dfrac{1}{2}$

$\dfrac{2}{4}$

$\dfrac{3}{6}$

3 Fractions equivalent to $\dfrac{2}{3}$

$\dfrac{2}{3}$

$\dfrac{4}{6}$

$\dfrac{6}{9}$

3 Fractions equivalent to $\dfrac{3}{4}$

$\dfrac{3}{4}$

$\dfrac{6}{8}$

$\dfrac{9}{12}$

48. $\dfrac{2}{3} > \dfrac{1}{2}$

$\dfrac{2}{3}$

$\dfrac{1}{2}$

$\dfrac{5}{8} < \dfrac{3}{4}$

$\dfrac{5}{8}$

$\dfrac{3}{4}$

49. $2 \div \dfrac{2}{3}$

2

| K | K | | K | K | |
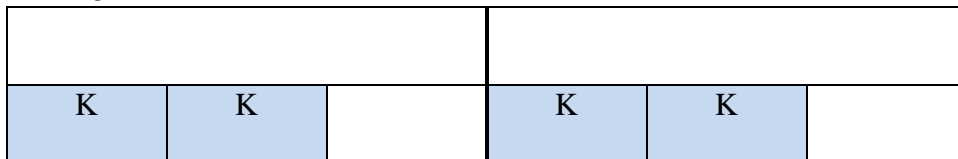
Divide each bar
into 3 and keep 2
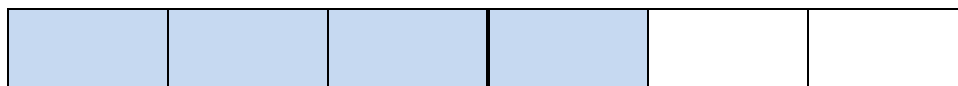
Re-arrange.
Answer is 4/3

Using the measurement model of division.

SOLUTIONS: Math 211 Final Practice Problems, Page 9

50. Three tired and hungry people had a bag of apples. While the other two were asleep, one of the three awoke, ate one-third of the apples, and went back to sleep. Later a second person awoke, ate one-third of the remaining apples, and went back to sleep. Finally, the third person awoke and ate one-third of the remaining apples, leaving 8 apples in the bag. How many apples were in the bag originally?
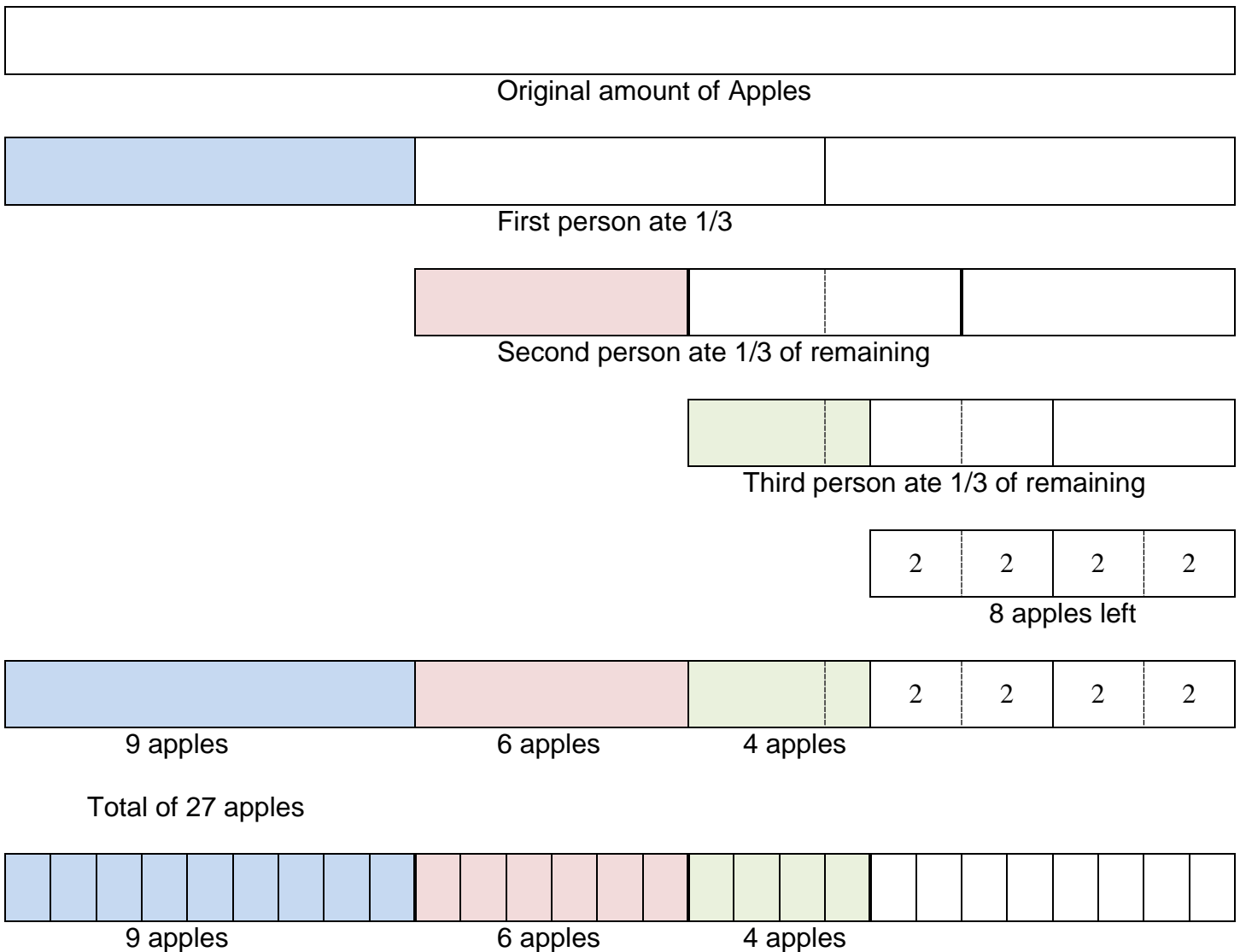
Original amount of Apples

First person ate 1/3

Second person ate 1/3 of remaining

Third person ate 1/3 of remaining

2	2	2	2

8 apples left

2	2	2	2

9 apples   6 apples   4 apples

Total of 27 apples

9 apples   6 apples   4 apples

Three tired and hungry people had a bag of apples. While the other two were asleep, one of the three awoke, ate one-fourth of the apples, and went back to sleep. Later a second person awoke, ate one-fourth of the remaining apples, and went back to sleep. Finally, the third person awoke and ate one-fourth of the remaining apples, leaving 27 apples in the bag. How many apples were in the bag originally?

Working backwards: There were 27 apples left in the bag at end;
The third person ate one-fourth of the remaining apples, leaving three-fourths. So, the three-fourths left is equal to 27.
Thus, one-fourth is $27 \div 3 = 9$ apples. The third person ate 9 apples.
Before he ate the 9 apples, there were 27+9=36 apples.
The second person ate one-fourth of the remaining apples, leaving three-fourths. So, the three-fourths left is equal to 36.
Thus, one-fourth is $36 \div 3 = 12$. The second person ate 12 apples.

Before he ate the 12 apples, there were 36+12=48 apples.
The first person ate one-fourth of the apples, leaving three-fourths. So, the three-fourths is equal to 48.
Thus, one-fourth is 48÷3=16. The first person ate 16 apples.
Before he ate the 16 apples, there were 48+16=64 apples.
There were 64 apples in the bag originally

51. Your father gives one-half of the money in his pocket to your mother, one-fourth of what is left to your brother, and one-third of what then remains to your sister. He then splits the rest with you. If you get $2, how much did your father start with?

All the money in father's wallet

Mom							

Half the money goes to Mom

Mom			Brother				

¼ of what is left to Brother

Mom			Brother	Sister		

1/3 of what is left to Sister

Mom			Brother	Sister	Dad	You $2

Splits rest with you; you get $2

Mom $2	$2	$2	$2	Brother $2	Sister $2	Dad $2	You $2

There was a total of $16 in father's wallet at the start.

Extension: If mother receives 1/3

All the money in father's wallet

Mom		

Mom gets 1/3 of the money

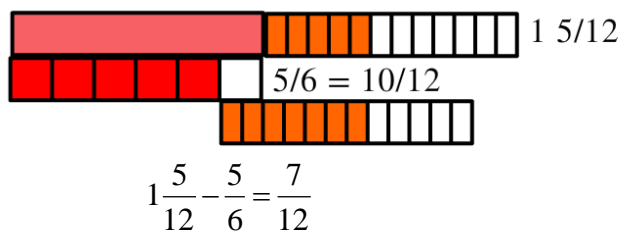| Mom $4		Brother $2	Sister $2	Dad $2	You $2

There was a total of $12 in father's wallet

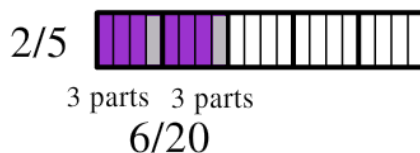52. Fractions bars to illustrate sums, differences, products and quotients of fractions.
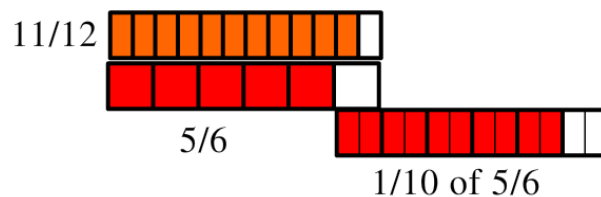
a. $\dfrac{2}{3}+\dfrac{1}{4}$

$2/3 = 8/12$

$1/4=3/12$

$2/3+1/4=11/12$

b. $1\dfrac{5}{12}-\dfrac{5}{6}$

$1\ 5/12$

$5/6 = 10/12$

$1\dfrac{5}{12}-\dfrac{5}{6}=\dfrac{7}{12}$

c. $\dfrac{3}{4}\times\dfrac{2}{5}$

$2/5$

3 parts 3 parts

$6/20$
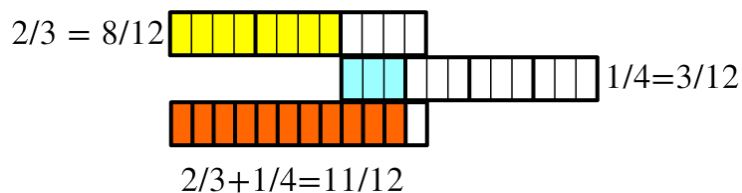
Sentence: $\dfrac{3}{4}\times\dfrac{2}{5}=\dfrac{6}{20}$

d. $\dfrac{11}{12}\div\dfrac{5}{6}$

$11/12$

$5/6$

$1/10$ of $5/6$

Sentence: $\dfrac{11}{12}\div\dfrac{5}{6}=1\dfrac{1}{10}$